Neoclassical orbit calculations with a full-f code for tokamak edge plasmas[*]

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Presented at the
APS-DPP Annual Meeting
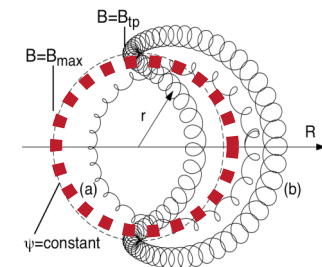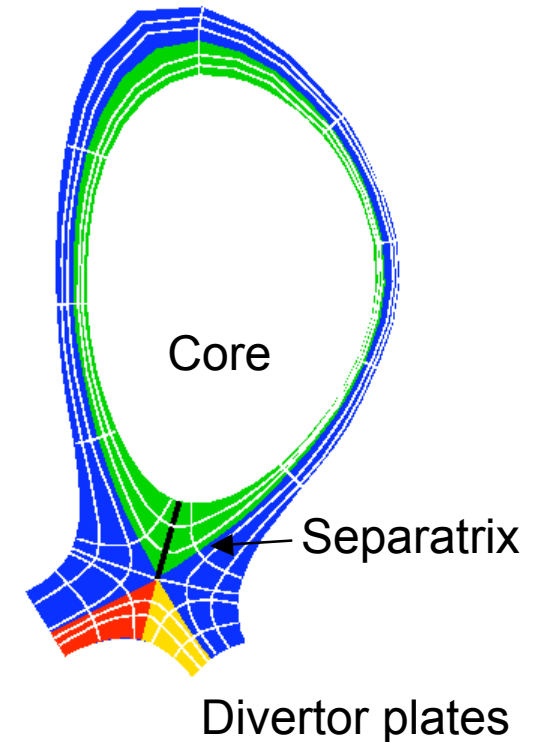Nov. 17-21, 2008
Dallas, TX

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Outline

- **Goal and overview of Edge Simulation Laboratory codes**

- **TEMPEST prototype 4D and 5D code applied to circular and divertor geometries**
  - reported here and by Xu this meeting; April '08 ITPA; Xu et al., PRL, Phys. Rev. E '08

- **Next generation 4D (--> 5D) code that builds on experience with TEMPEST, EGK, and NEO**
  - status reported here

Goal: gyrokinetic 4D and 5D continuum edge simulations for pedestal and SOL physics

$$\frac{\partial F_\alpha}{\partial t} + \bar{v}_d \cdot \nabla_\perp F_\alpha + (\bar{v}_{\parallel\alpha} + v_{Banos})\nabla_\parallel \partial F_\alpha$$

$$+ \left[q\frac{\partial \langle \Phi_0 \rangle}{\partial t} + \bar{\mu}\frac{\partial B}{\partial t} - \frac{qB}{B^*}\bar{v}_\parallel \nabla_\parallel \langle \delta\phi \rangle - q v_d^0 \cdot \bar{\nabla}\langle \delta\phi \rangle \right] \frac{\partial F_\alpha}{\partial E_0}$$

$$= C(F_\alpha, F_\alpha),$$

- **GK F-equation discretized with high order (4th); Fokker-Planck collisions; here energy-dependent Lorentz op.**

- **Electrostatic potential via <u>Poisson eqn</u>**

- **Circular & divertor geometry**

- **Runnable as**
  - 4-D for transport with F(Ψ,θ,ε,μ), or
  - 5-D for turbulence with F(Ψ,θ,ϕ ,ε,μ) - underway

- **Extensions planned:**
  - sources/sinks
  - model transport coefficients for initial anomalous transp.
  - generalized GK equations (see Qin; Dimits)
  - *field-aligned coordinates for evolving B


Core

Separatrix

Divertor plates
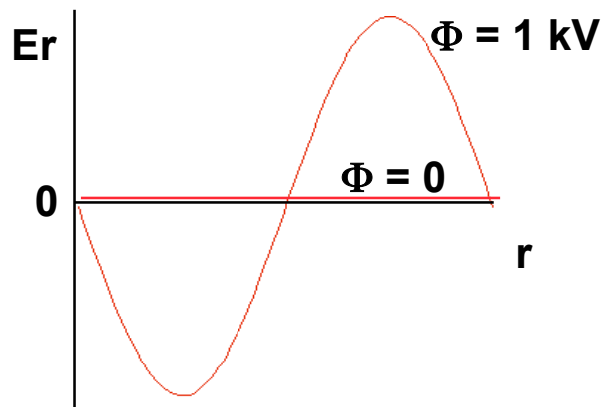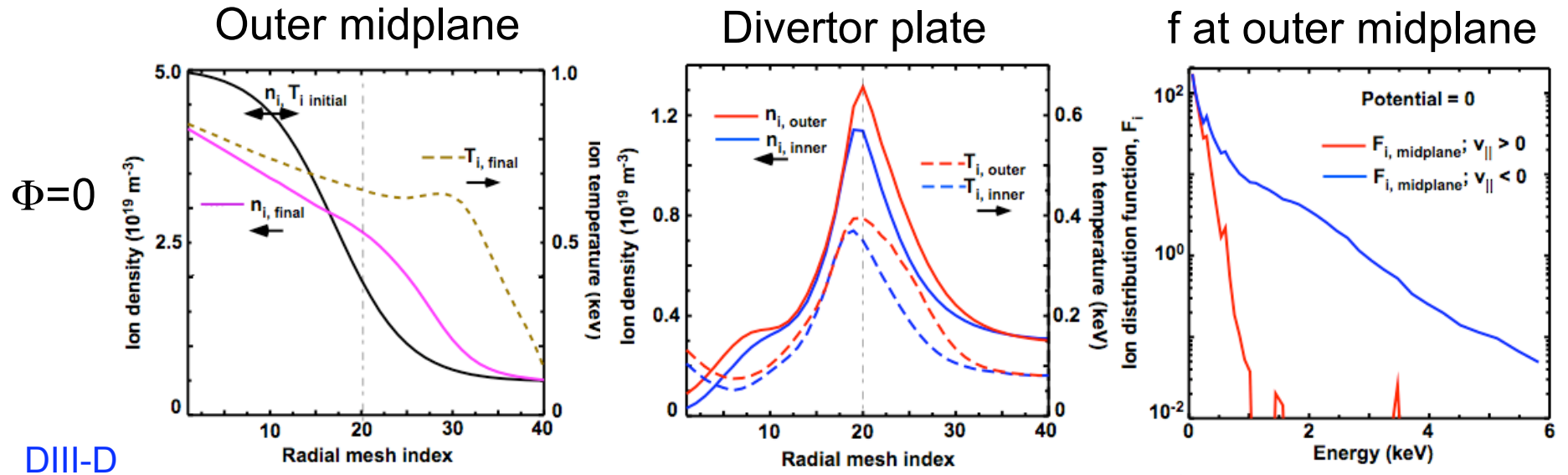
Edge Simulation Laboratory: code history

- **TEMPEST prototype 4D and 5D code applied to circular and divertor geometries**
  - reported here and by Xu this meeting; Xu et al., PRL, Phys. Rev. E '08

- **Next generation 4D (--> 5D) code that builds on experience with TEMPEST, EGK, and NEO**
  - reported here

- **EGK and NEO prototype flux-tube codes**
  - Invited talk, this APS-DPP meeting

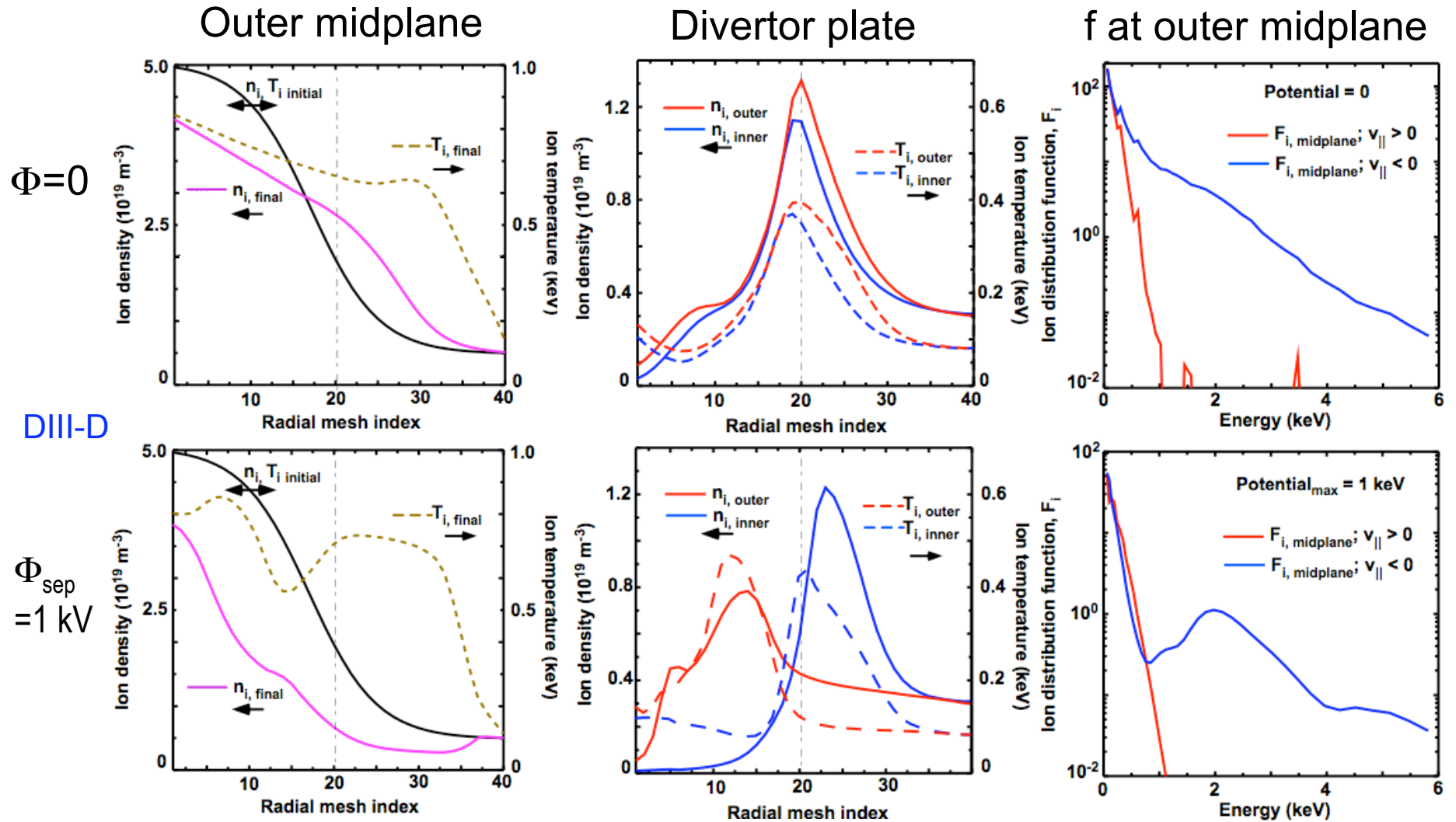# TEMPEST simulates kinetic ion profiles/ transport for a DIII-D magnetic equilibrium

- To clarify details of the results, the thin annular edge region is displayed in a convenient slab image

- Nevertheless, computations include full toroidal geometry effects
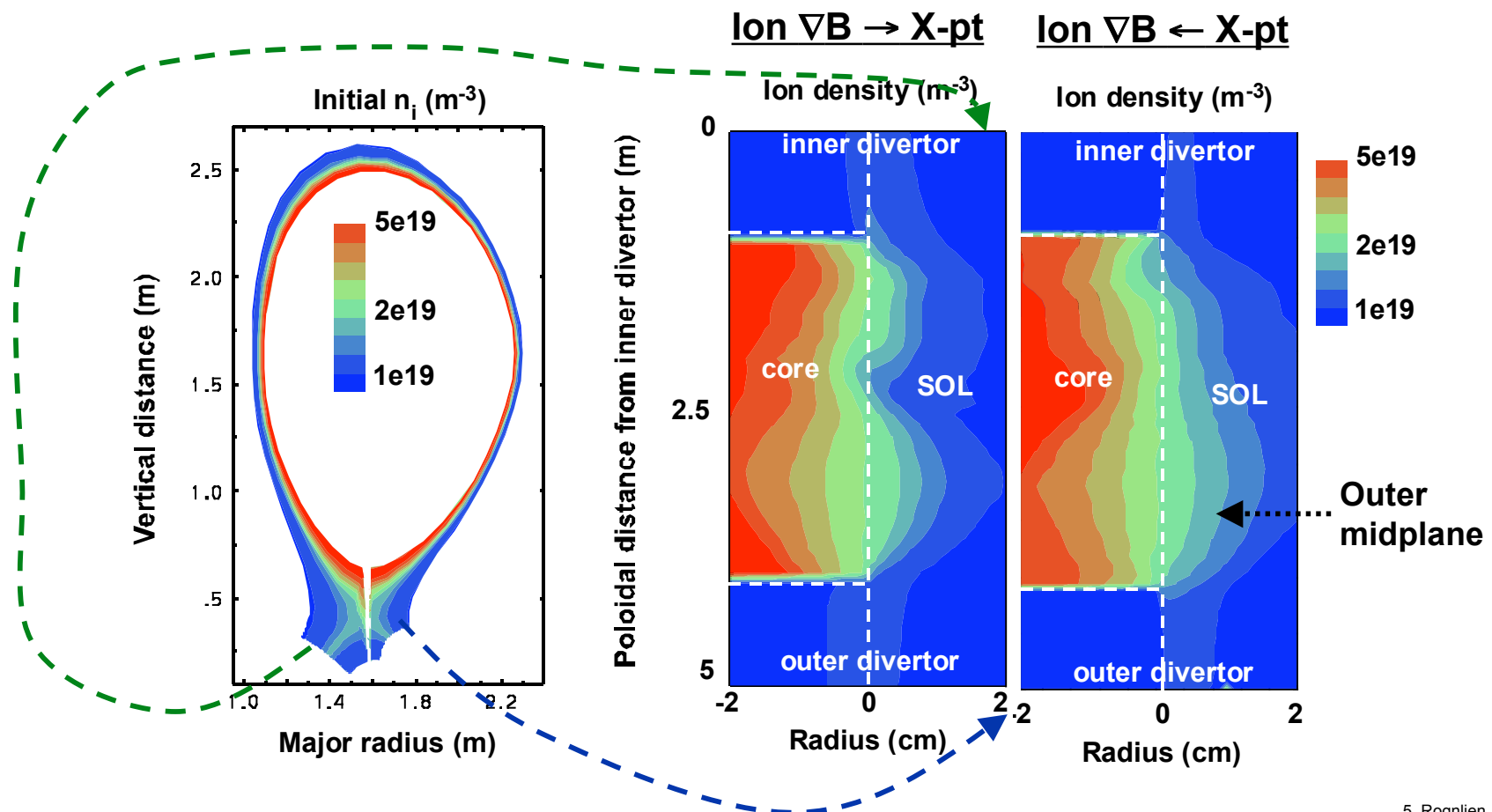
Ion ∇B → X-pt

Ion ∇B ← X-pt

Initial n_i (m^{-3})

Ion density (m^{-3})

Ion density (m^{-3})

TEMPEST simulations in divertor geometry with model E_r indicate expected asymmetries and non-Maxwellian f

edge Simulation Laboratory

Outer midplane

$\Phi=0$

DIII-D

Divertor plate

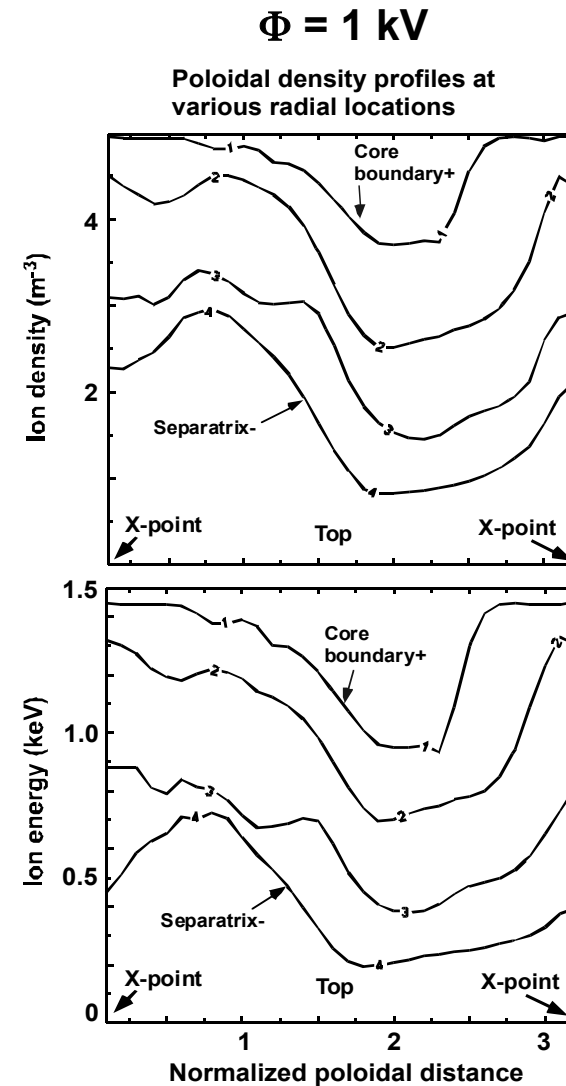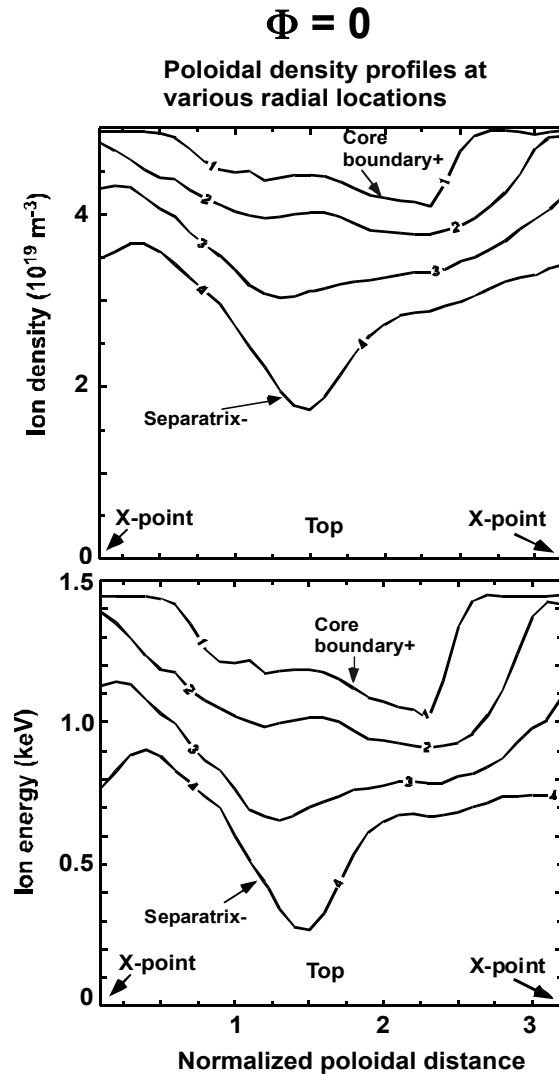## f at outer midplane

E_r

$\Phi = 1\ kV$

$\Phi = 0$

0

r

TEMPEST simulations in divertor geometry with model E_r indicate expected asymmetries and non-Maxwellian f

Outer midplane

Divertor plate

f at outer midplane

$\Phi = 0$

DIII-D

$\Phi_{sep} = 1$ kV

Pedestal profiles of n_i and T_i show strong poloidal asymmetries - even with E_r

$\Phi = 0$

Poloidal density profiles at various radial locations

$\Phi = 1$ kV

Poloidal density profiles at various radial locations

Steep-gradient case illustrates need for high-order advection scheme

Positive F for $v_\parallel > 0$

$t_n = 0$

Outer midplane

Bottom

Poloidal index

$E = 4.5$ keV
$Mu = 0.53$ keV/T
$E_\perp = Mu \times (B/B_{min})$
$B_{min} = 1.17$ T
$R_0 = 1.71$ m
$t_n = t\, v_{ti}/R$,
$v_{ti}/R = 1.83 \times 10^5$ s^{-1}

Negative F for $v_\parallel > 0$
[abs(F) - same color-bar]

$t_n = 0.82$

$t_n = 2.10$

Tanh profiles for density and temperature

$n_{i0} = 5 \times 10^{19}$ m^{-3}
$T_{i0} = 1$ kev

Normalized profiles n_i/n_{i0}, T_i/T_{i0}

Symmetry point for tanh(r)

Radial cell index (uniform mesh)

$a = 67.5$ cm

$\epsilon = a/R_0 = 0.395$

8.6 cm

Vertical position, Z (m)

Major radius, R (m)

$-B_{tor}$

B_{pol}

Magnetic field (T)

Major radius, R (m)

$-B_{tor}$ is into poloidal plane
$\otimes$

Ion grad-B drift

Z

R

Developing algorithms and next generation code to deal with the challenges of edge GK

Experience with TEMPEST, EGK, and NEO emphasize needs:

- **Conservation** ------------------>
- **Low-dissipation advection (orbits)**
- **Preservation of distribution function positivity** ----------->
- **Efficient resolution of a large and complicated phase space**
- **Use $f(v_{||},\mu)$ or $f(v_{||},\theta_{pa})$, not $f(E,\mu)$**
- **Robust for high anisotropy**

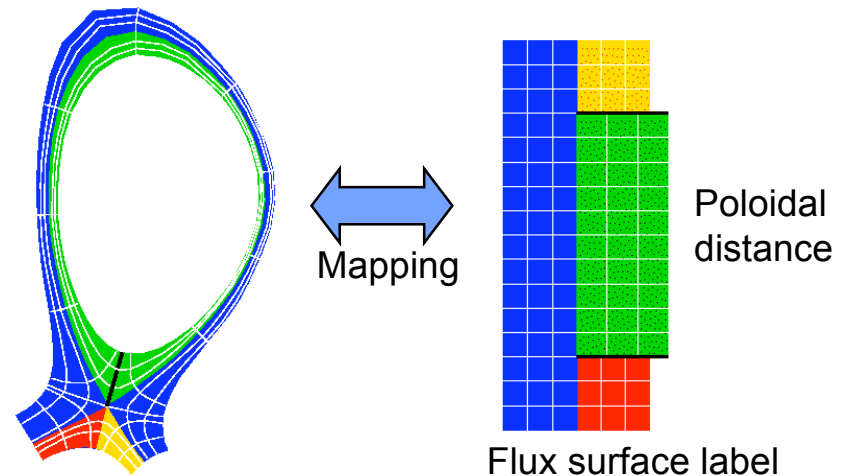*Numerical methodologies:*

- **Finite volume discretizations with conservative formulations**
- **High-order (4th) discretization**
- **Mapped, multiblock grids**
- **Data structures allow parallization in both configuration & velocity space**

Applied math participants:
M. Dorr, J. Hittinger, LLNL
P. Colella, D. Martin, LBNL

Mapping

Poloidal distance

Flux surface label

Consistent 4th order treatment of geometry and function variations on _uniform_ mapped mesh

Have developed high-resolution discretizations and are applying to gyrokinetic Vlasov equation

- **Gyrokinetic Vlasov equation describes advection by a phase space velocity field that is a nonlocal function of the distribution function f :**

$$\frac{\partial f}{\partial t} + \boldsymbol{\nabla}_{\boldsymbol{R}} \cdot \left(\dot{\boldsymbol{R}}(f)f \right) + \frac{\partial}{\partial v_{\parallel}} \left(\dot{v}_{\parallel}(f)f \right) = 0$$

- **Dependence of the phase space velocities $\dot{R}$ and $\dot{v}_{\parallel}$ on f is through the Poisson solve**

- **To obtain a high-order discretization that is robust for this highly nonlinear system, we combine**

  – fourth-order flux-corrected transport (FCT) spatial discretization: as part of this project - Colella and Sekora, JCP '08

    - fourth-order accuracy where solution is smooth (does not reduce accuracy at smooth extrema like classical FCT and PPM)
    - combined with an FCT limiter (Zalesak) preserves positivity of **f**

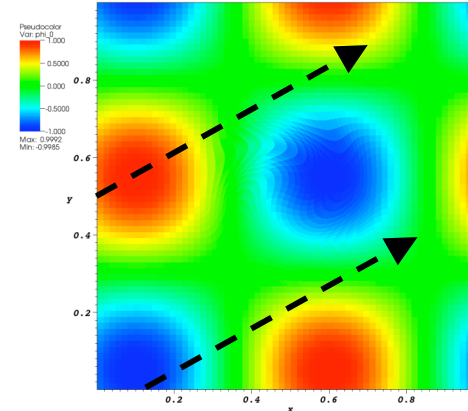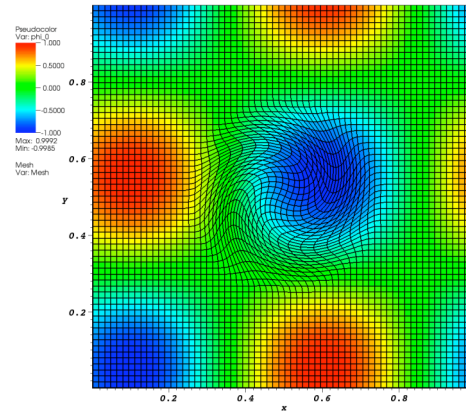# Temporal convection of 2D "blobs and holes" show robustness of new method
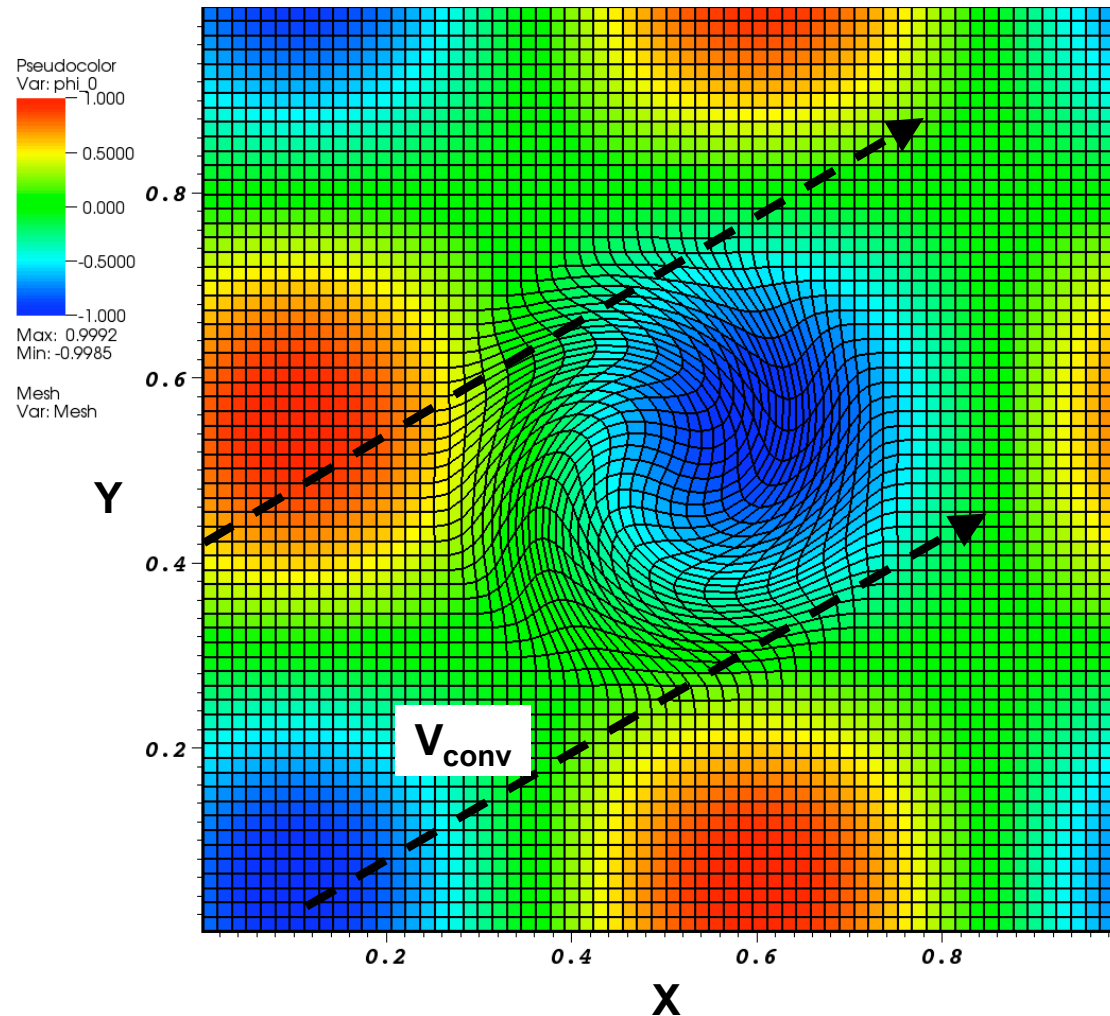
Application of the mapped grid finite volume discretization to an advection test problem
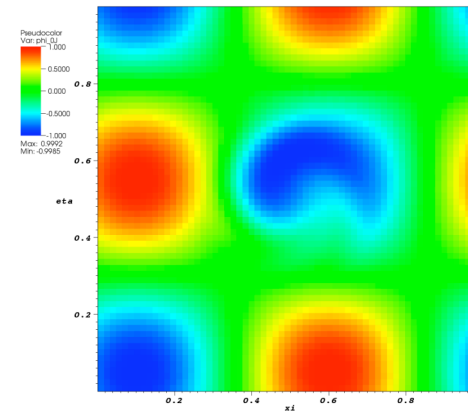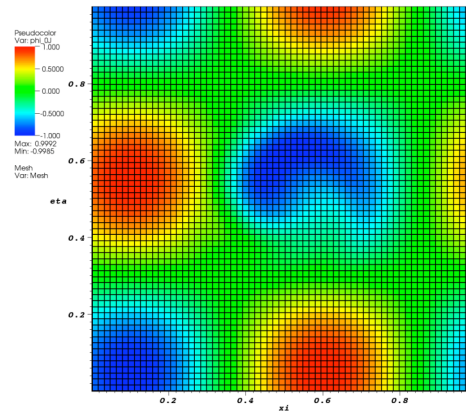
Physical coordinates: highly irregular mesh in center

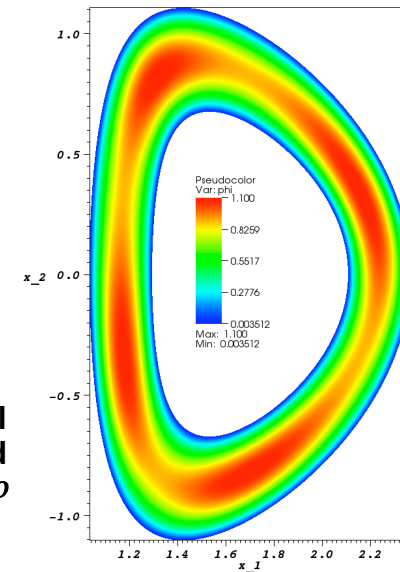MappedAdvect.mov

Mapped to uniform computational coordinates

Fourth order mesh convergence demonstrated

4th-order accuracy GK Poisson discretization obtained on core equilibrium geometries
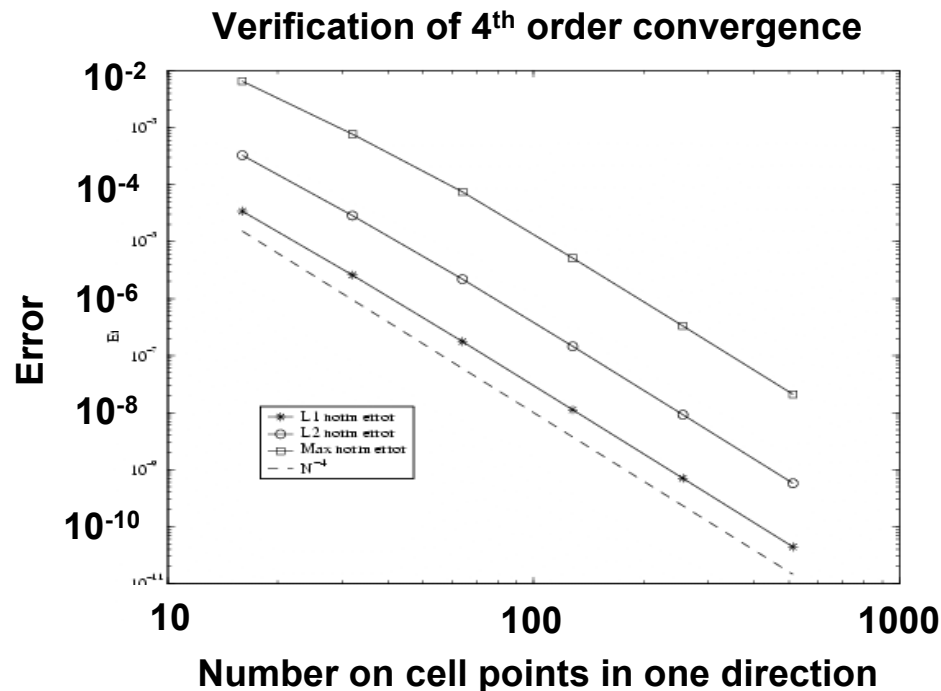
Demonstrate accuracy: Given Φ, use quadrature to manufacture ρ such that

$$\nabla \cdot \left\{ \left[\epsilon_0 I + \frac{n_i}{B^2} \left(I - \vec{b}\vec{b}^T \right) \right] \nabla \Phi \right\} = \rho$$

prescribed density profile n_i and a magnetic field from an analytically Miller equilibrium model

Verification of 4th order convergence

Error

L1 notm error
L2 notm error
Max notm error
N^{-4}

Number on cell points in one direction

Potential computed from exact ρ

Pseudocolor
Var: phi
1.100
0.8259
0.5517
0.2776
0.003512
Max: 1.100
Min: 0.003512

Convergence of *Hypre* CG solver preconditioned with multigrid solution of second-order operator

iter	Relative residual
1	6.62e-03
2	1.19e-03
3	2.24e-04
4	1.17e-04
5	4.59e-05
6	1.18e-05

Schedule for next year:

- **New ESL code released to physics team:**
    - Now completing 4D, core: results by Dec
    - 4D, divertor: spring '09
    - 5D, core: summer-fall '09
    - 5D, divertor, start summer '09

Summary

1. **Kinetic ion orbit effects produce substantial poloidal variations in n_i and T_i ; E_r via orbit squeezing can reduce variation, but orbit loss can also be influential**

2. **New ESL code components being assembled/tested**
   - Advection - tested
   - Poisson eqn -tested
   - 4D version with drift orbits nearly assembled